University of Sheffield

Reach unified channel characteristics for the transverse advection -dispersion equation

40th International School of Hydraulics
Kąty Rybackie, Poland

A WORLD TOP 100 UNIVERSITY

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23-26 May 2023

The Advection-Dispersion Equation

The ADE:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = D_x\frac{\partial^2 c}{\partial x^2} + D_y\frac{\partial^2 c}{\partial y} + D_z\frac{\partial^2 c}{\partial y^2}$$

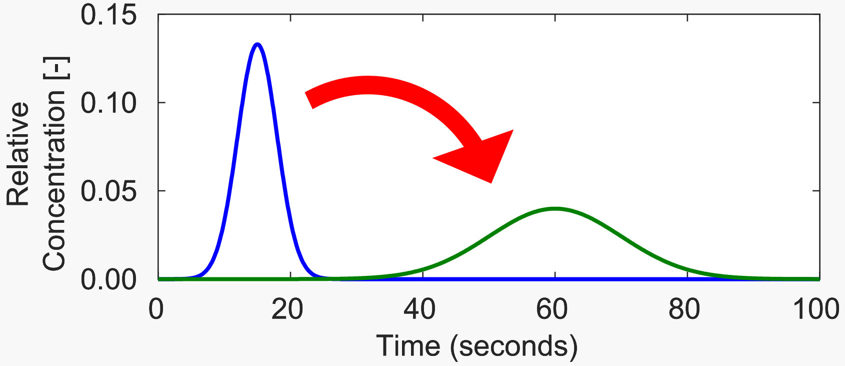
Typically simplified, e.g., routing solutions:

$$c(x_2, t) = \int_{\gamma=-\infty}^{\infty} \frac{c(x_1, \gamma)U}{\sqrt{4\pi D_x \bar{t}}} \exp\left[-\frac{U^2(\bar{t} - t + \gamma)^2}{4D_x \bar{t}}\right] d\gamma$$

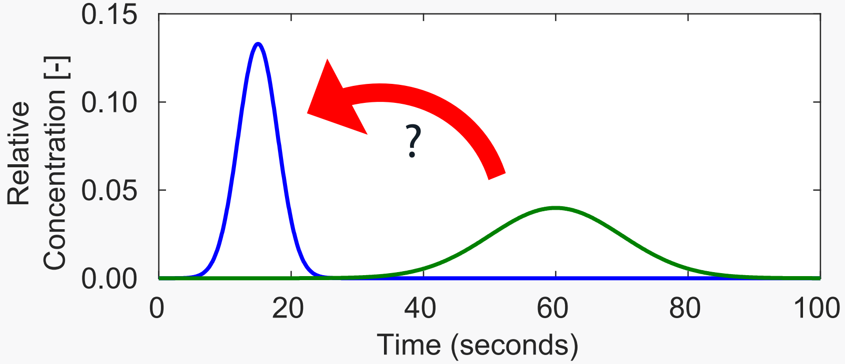
$$c(x_2, y) = \int_{\lambda=-\infty}^{\infty} \frac{c(x_1, \lambda)}{\sqrt{4\pi D_y \bar{t}}} \exp\left[-\frac{(\lambda - y + V\bar{t})^2}{4D_y \bar{t}}\right] d\lambda$$

Two types of applications:

Used for prediction in modelling

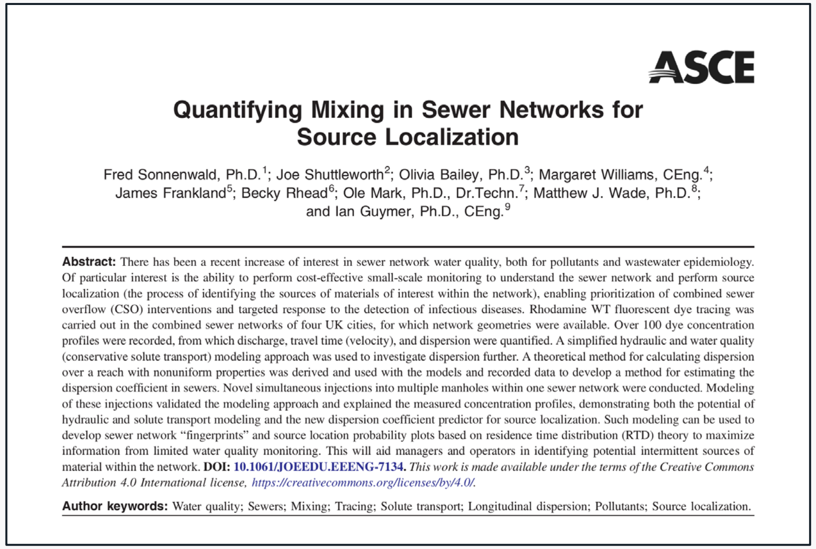


Used analytically to determine dispersion coefficients (regression)



Longitudinal dispersion in sewers

- Conducted dye tracing in sewers
- Attempting to link sewer hydraulics to D_x
- Challenge: changing pipe diameters
- When performing regression, what are the correct conduit characteristics?



ASCE

Quantifying Mixing in Sewer Networks for Source Localization

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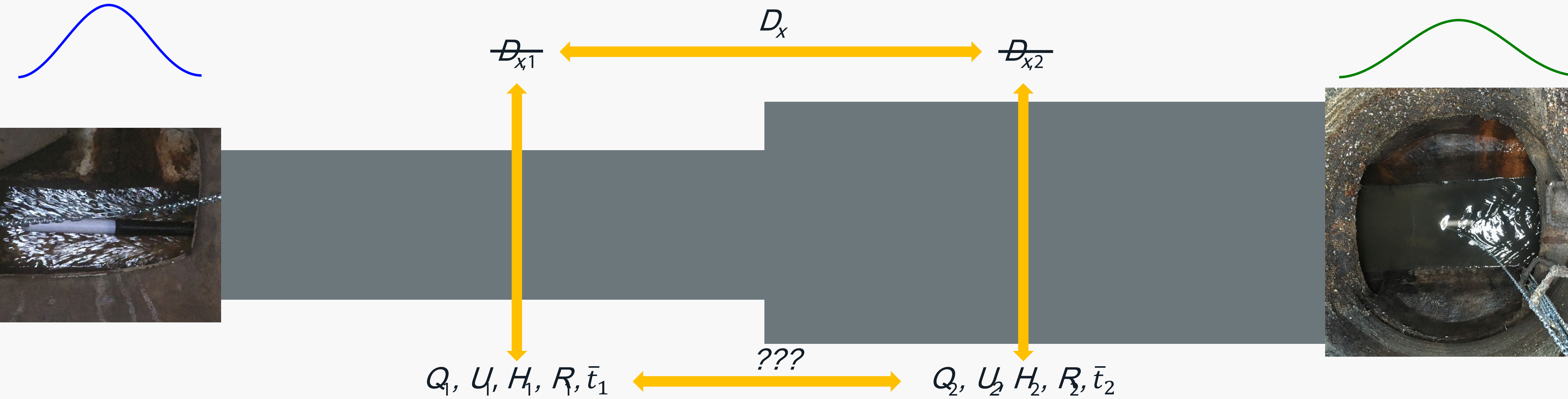
Abstract: There has been a recent increase of interest in sewer network water quality, both for pollutants and wastewater epidemiology. Of particular interest is the ability to perform cost-effective small-scale monitoring to understand the sewer network and perform source localization (the process of identifying the sources of materials of interest within the network), enabling prioritization of combined sewer overflow (CSO) interventions and targeted response to the detection of infectious diseases. Rhodamine WT fluorescent dye tracing was carried out in the combined sewer networks of four UK cities, for which network geometries were available. Over 100 dye concentration profiles were recorded, from which discharge, travel time (velocity), and dispersion were quantified. A simplified hydraulic and water quality (conservative solute transport) modeling approach was used to investigate dispersion further. A theoretical method for calculating dispersion over a reach with nonuniform properties was derived and used with the models and recorded data to develop a method for estimating the dispersion coefficient in sewers. Novel simultaneous injections into multiple manholes within one sewer network were conducted. Modeling of these injections validated the modeling approach and explained the measured concentration profiles, demonstrating both the potential of hydraulic and solute transport modeling and the new dispersion coefficient predictor for source localization. Such modeling can be used to develop sewer network "fingerprints" and source location probability plots based on residence time distribution (RTD) theory to maximize information from limited water quality monitoring. This will aid managers and operators in identifying potential intermittent sources of material within the network.

DOI: 10.1061/JOEEDU.EEENG-7134.

Author keywords: Water quality; Sewers; Mixing; Tracing; Solute transport; Longitudinal dispersion; Pollutants; Source localization.

https://doi.org/10.1061/JOEEDU.EEENG-7134



Reach Unification

Averaging method to combine the effects of each sub-reach, by considering virtual intermediate measurement locations

$$c(x_2,t) = \int_{\gamma=-\infty}^{\infty} \frac{c(x_1,\gamma)U}{\sqrt{4\pi D_x \bar{t}}} \exp\left[-\frac{U^2(\bar{t}-t+\gamma)^2}{4D_x\bar{t}}\right] d\gamma$$

$$c(x_2,t) = \int_{\gamma=-\infty}^{\infty} \frac{c(x_1,\gamma)U_1}{\sqrt{4\pi D_{x,1} \bar{t}_1}} \exp\left[-\frac{U_1^2(\bar{t}_1-t+\gamma)^2}{4D_{x,1}\bar{t}_1}\right] d\gamma$$

$$c(x_3,t) = \int_{\tau=-\infty}^{\infty} \frac{c(x_2,\tau)U_2}{\sqrt{4\pi D_{x,2} \bar{t}_2}} \exp\left[-\frac{U_2^2(\bar{t}_2-t+\gamma)^2}{4D_{x,2}\bar{t}_2}\right] d\tau$$

$$c(x_3,t) = \int_{\tau=-\infty}^{\infty} \frac{\int_{\gamma=-\infty}^{\infty} \frac{c(x_1,\gamma)U_1}{\sqrt{4\pi D_{x,1}\bar{t}_1}} \exp\left[-\frac{U_1^2(\bar{t}_1-t+\gamma)^2}{4D_{x,1}\bar{t}_1}\right] d\gamma \; U_2}{\sqrt{4\pi D_{x,2}\bar{t}_2}} \exp\left[-\frac{U_2^2(\bar{t}_2-t+\tau)^2}{4D_{x,2}\bar{t}_2}\right] d\tau$$

$$U = \sum_{i=1}^{N} U_i \bar{t}_i \,/\, \sum_{i=1}^{N} \bar{t}_i$$

$$D_x = \frac{\left(\sum_{i=1}^{N} U_i \bar{t}_i\right)^2 \sum_{i=1}^{N}\left(D_{x,i}\bar{t}_i \prod_{j=1,j\neq i}^{N} U_j^2\right)}{\left(\prod_{i=1}^{N} U_i^2\right)\left(\sum_{i=1}^{N} \bar{t}_i\right)^3}$$

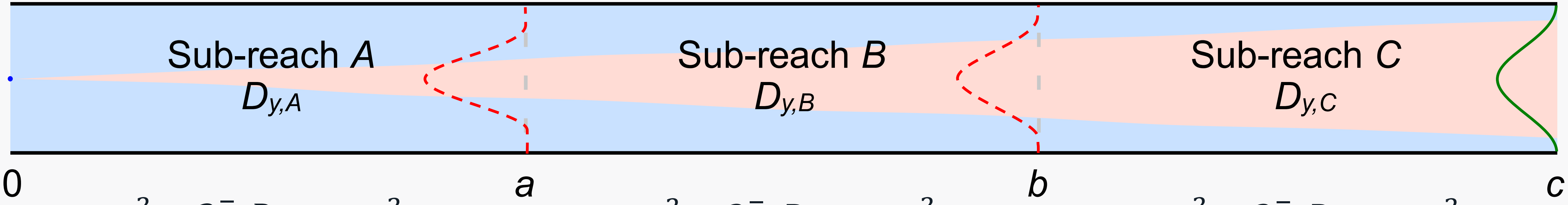
$$D_x = \alpha U \qquad \alpha = U \frac{\sum_{i=1}^{N} \alpha_i \bar{t}_i / U_i}{\sum_{i=1}^{N} \bar{t}_i}$$

Transverse Reach Unification

- Transverse mixing is of interest near outfalls, etc.
- *River Mixing* (Rutherford, 1994) suggests length weighted averaging of D_y
  - Is this appropriate given what we know about longitudinal reach unification?

SG Wallis suggested rearrangement and substitution of the method of moments, relating variance and D_y

$$D_y = \frac{U}{2}\frac{(\sigma_c^2 - \sigma_0^2)}{(x_c - x_0)}$$



$$\sigma_0^2 + 2\bar{t}_A D_{y,A} = \sigma_a^2 \qquad \sigma_a^2 + 2\bar{t}_B D_{y,B} = \sigma_b^2 \qquad \sigma_b^2 + 2\bar{t}_C D_{y,C} = \sigma_c^2$$

$$\bar{t}_A = (x_a - x_0)/U_A \qquad \bar{t}_B = (x_b - x_a)/U_B \qquad \bar{t}_C = (x_c - x_b)/U_C$$

$$U = \frac{\sum_{i=1}^{N} U_i \bar{t}_i}{\sum_{i=1}^{N} \bar{t}_i} \qquad D_y = \frac{\sum_{i=1}^{N} D_{y,i} \bar{t}_i}{\sum_{i=1}^{N} \bar{t}_i}$$

Not length weighted!

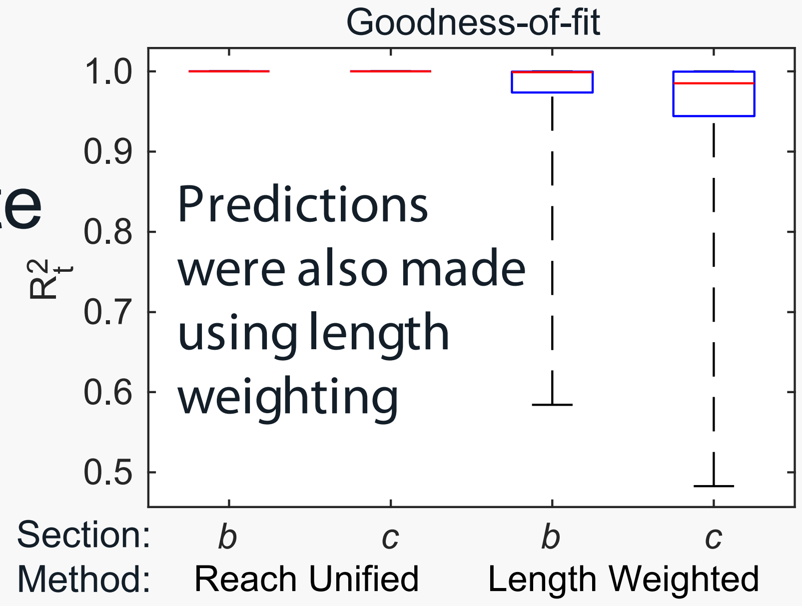
$$D_y = \alpha U \qquad \alpha = \frac{\sum_{i=1}^{N} \alpha_i \Delta x_i}{\sum_{i=1}^{N} \Delta x_i}$$

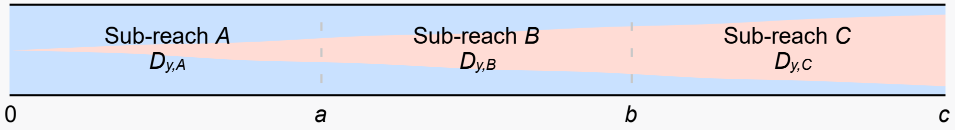
Length weighted!

Not the same equations as for D_x

Validation

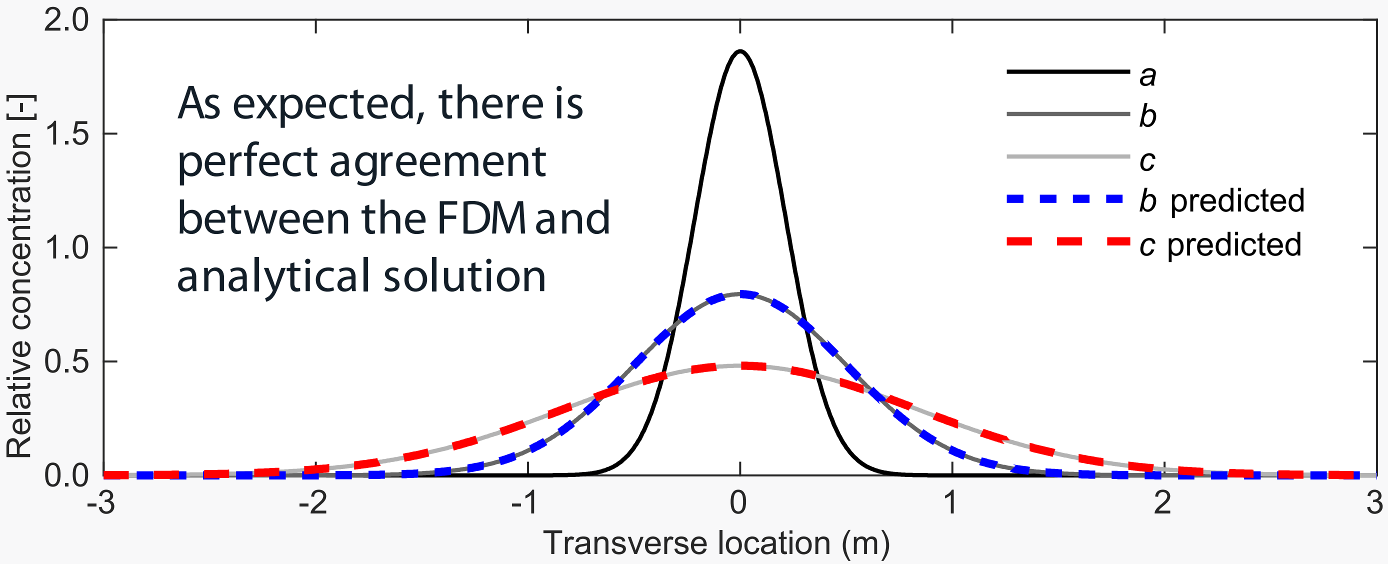
- 100 synthetic data were generated using a finite differences model of the 2D depth-averaged ADE and the relationship $D_y = 0.13U\sqrt{f/8}$
- Each sub-reach had a different length, velocity, depth, and friction
- Predictions were made with a direct solution and compared





$\Delta x_A = 5$ m	$\Delta x_B = 10$ m	$\Delta x_C = 15$ m
$U_A = 0.500$ m/s	$U_B = 0.250$ m/s	$U_C = 0.125$ m/s
$H_A = 1$ m	$H_B = 2$ m	$H_C = 4$ m
$f_A = 0.01$	$f_C = 0.05$	$f_C = 0.10$

etc.



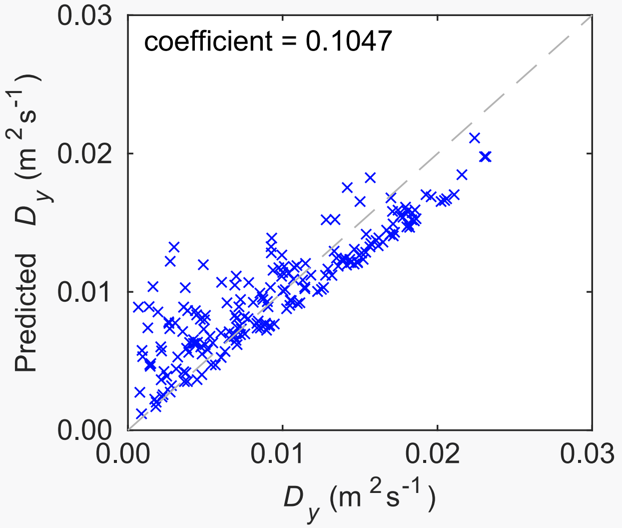
Effect on regression

- Recovering the slope coefficient $k = 0.13$ in $D_y = kU\sqrt{f/8}$
- U and f and averaged in different ways then least squares fit to D_y to obtain k

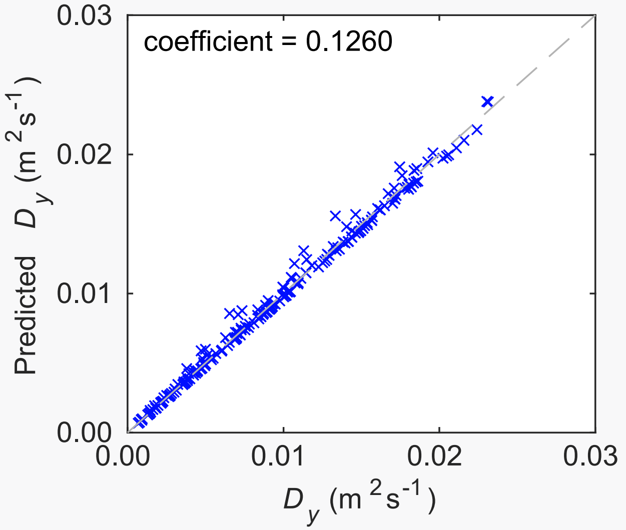
When applying reach unification, let $\alpha = \sqrt{f/8}$, i.e.,

$$\alpha = \frac{\sum_{i=1}^{N} \alpha_i \Delta x_i}{\sum_{i=1}^{N} \Delta x_i} \rightarrow \sqrt{f/8} = \frac{\sum_{i=1}^{N} \sqrt{f_i/8}\Delta x_i}{\sum_{i=1}^{N} \Delta x_i}$$

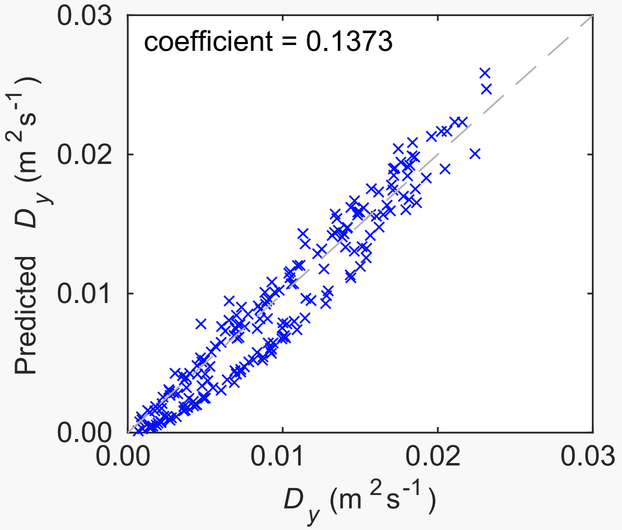
Length weighted U and f



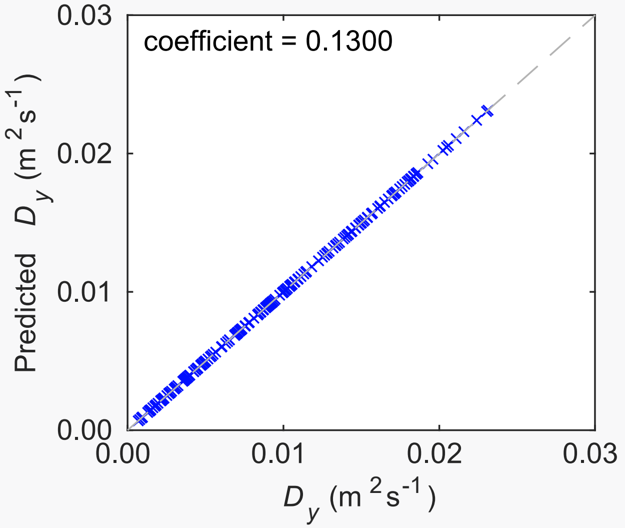
Travel time weighted U and length weighted f



Travel time weighted U and f



Reach Unification



Conclusions

- Reach unification incorporates sub-reach characteristics into the equivalent single reach values
  - Dispersion coefficients or depth, etc.
- This allows for direct analytical downstream predictions in channels with longitudinally varying characteristics
- It also refines comparison between experimentally obtained dispersion coefficients and channel characteristics
- The appropriate reach unification changes between the longitudinal and transverse ADE

Thanks for listening! Questions?